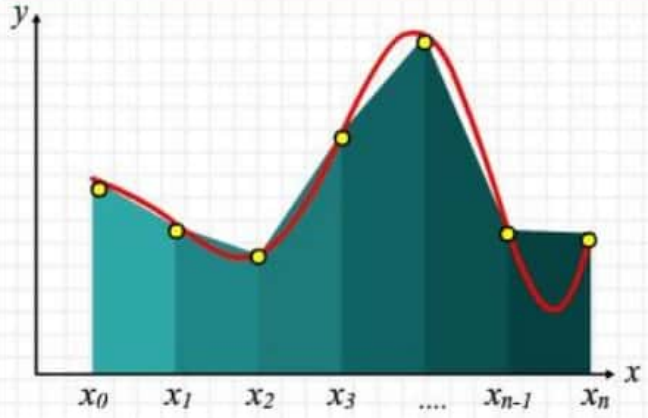
AREA UNDER THE CURVE

Trapezoidal Riemann Approximation



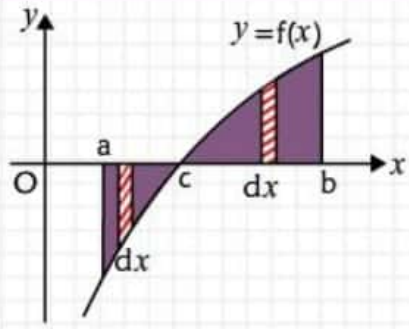
$$\text{Area} = \sum_{i=1}^{n} \frac{f(x_{i-1}) + f(x_i)}{2} \Delta x \quad ; \Delta x = \frac{b-a}{n}$$

Riemann Sum:

$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{f(x_{i-1}) + f(x_i)}{2} \Delta x$$

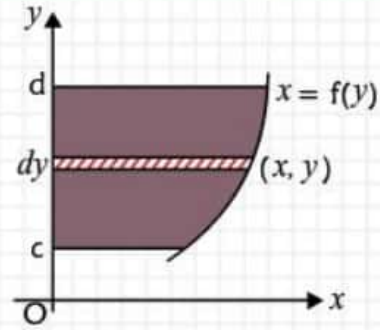
$$= \int_a^b f(x)dx \qquad dx - \text{infinitely small}$$

Area By Vertical Strips



$$A = \left| \int_a^c f(x)dx \right| + \int_c^b f(x)dx$$

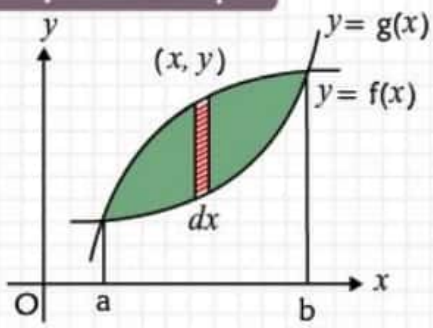
Area By Horizontal Strips



$$A = \int_c^d f(y)dy = \int_c^d x\,dy$$

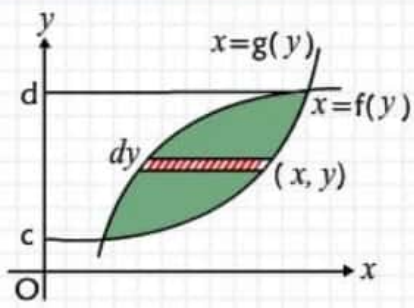
Area Enclosed Between Two Curves

Case – I : By vertical strips



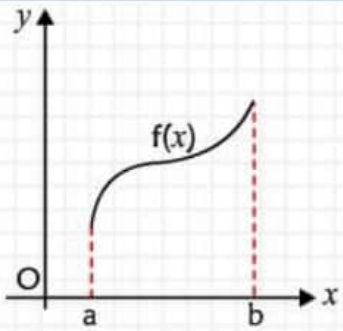
$$A = \int_a^b f(x)dx - \int_a^b g(x)dx = \int_a^b [f(x) - g(x)]dx$$

Case – II : By horizontal strips



$$A = \int_c^d f(y)dy - \int_c^d g(y)dy = \int_c^d [f(y) - g(y)]dy$$

Average Value of a Function



$$y = f(x) \; ; \; a \le x \le b$$

$$y_{avg} = \frac{1}{b-a} \int_c^d f(x)dx$$

Useful Results

- Whole area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab (units)^2.
- Area enclosed between the parabola $y^2 = 4ax$ & $x^2 = 4by$ is $\frac{16\,ab}{3}$ (units)^2.
- Area enclosed between the parabola $y^2 = 4ax$ & $y = mx$ is $\frac{8\,a^2}{3m^3}$ (units)^2.